Intersectoral Linkages, Diverse Information, and Aggregate Dynamics

Manoj Atolia*
Florida State University

Ryan Chahrour†
Boston College

October 8, 2019

Abstract

What are the aggregate consequences of information frictions? We address this question in a multi-sector real business cycle model in which firms learn from market-based information. Theoretically, we present two distinct cases in which the aggregate effects of incomplete information completely disappear: either (i) market-based information reveals sector-level optimal actions, or (ii) market-based information reveals aggregate conditions in the economy. When the model is calibrated to United States' sectoral data, both conditions hold almost exactly and incomplete information has a negligible effect on aggregate dynamics.

Keywords: Imperfect information, Information frictions, Dispersed information, Sectoral linkages, Strategic complementarity, Higher-order expectations

JEL Codes: D52, D57, D80, E32

*Department of Economics, Florida State University, Tallahassee, FL 32306, U.S.A. Telephone: 850-644-7088. Email: matolia@fsu.edu.

†Corresponding author. Department of Economics, Boston College, Chestnut Hill, MA 02467, U.S.A. Telephone: 617-552-0902. Email: ryan.chahrour@bc.edu.

1 Introduction

The literature on information frictions in macroeconomics describes two main channels through which incomplete information may drive aggregate economic outcomes. First, peoples' correlated errors about local conditions may lead to aggregate fluctuations that exceed those warranted by economic fundamentals alone. Second, peoples' disagreement about the state of the economy, combined with an incentive to coordinate actions, may lead to delayed aggregate responses to shocks.

For either of these information channels to operate, economic agents must have different information from one another. But, why should peoples' information differ? A natural hypothesis is that information differs because people participate in different markets, and therefore observe different pieces of the economy. Differences in market participation, and the corresponding differences in information, are often founded in the literature by assuming agents inhabit informationally-isolated islands, which serve as a short-hand for deeper sources of informational heterogeneity.

In this paper, we examine one underlying source of such differences—the sparse nature of firms' input-output connections—and ask whether it can give rise to the sort of disagreement that is essential to information-based theories of the business cycle. We analyze this question in the context of a neoclassical model in which informational asymmetries are *market-generated*: Firms observe their own productivity, the price of their output, and the prices of those goods that are inputs in their production.[1] In this setup, the information held by agents depends directly on the pattern of input-output linkages in the economy. A long standing research agenda explores how intermediate production structures influence the propagation of sectoral shocks to the aggregate economy.[2] We study the role of such linkages in propagating information.

This environment is a natural place to look for the main information channels. First, the dependence of agents' information on endogenous variables—prices—leads to the potential for informational feedback effects. Other authors have demonstrated that such feedbacks can magnify initial errors made by agents.[3] Second, realistically sparse input-output structures imply that firms' information is significantly non-overlapping or dispersed, while sectoral linkages lead to complementarity in firms' investment.[4] These two channels—dispersed

[1]This assumption follows the suggestion of Hellwig and Venkateswaran (2009) and Graham and Wright (2010) that, when assessing the role of information frictions, firms' actions should be allowed to be conditioned *at least* on the prices that are directly relevant to their own choices.

[2]Papers along this line include Long and Plosser (1983), Horvath (1998), Dupor (1999), Horvath (2000), and Acemoglu et al. (2012).

[3]See Benhabib et al. (2015), Vives (2017), and Chahrour and Gaballo (2018) for examples.

[4]The term "dispersed information" is often used to describe the situation of atomistic agents, each of whose contribution to the aggregate is negligible. When necessary to distinguish between that situation

information and strategic complementarities—are also the focus of the new-Keynesian literature that uses information frictions to explain delayed responses to nominal shocks.[5] In our environment, the same forces could deliver gradual investment responses to fundamental shocks, offering an alternative to the investment adjustment costs and other frictions common in the DSGE literature.

Our central finding is that, once firms are allowed to learn from the prices emerging from the markets in which they participate, it is extremely difficult to generate a substantial impact of incomplete information. To provide intuition for this result, we use a simplified sectoral model to establish three themes that, together, suggest a limited role for the informational channels emphasized by the literature. These themes are (1) sectoral prices, regardless of input-output structure, are very informative about both local *and* aggregate conditions; (2) when incomplete information leads sectors to make suboptimal choices, knowledge of aggregate conditions leads sectoral mistakes to cancel out; and (3) even when information about aggregate conditions is incomplete, beliefs about those conditions tend to be the same across sectors. In short, regardless of sector level disagreement, the aggregate effects of dispersed information are virtually nil.

We analyze a standard (linearized) sectoral model in which sectoral productivity shocks are the only shocks hitting the economy and investment choices are made with incomplete information. We begin by providing a sufficient condition for the existence of an *aggregate representation* of the sectoral model *under full information*, a situation in which the aggregate dynamics of the economy are determined by a set of equations that are isomorphic to a single-sector real business cycle model. Our result subsumes Dupor (1999) and demonstrates that, for a broad class of economies, aggregate dynamics can be determined without reference to any sector-specific quantities or shocks. Our subsequent results regarding the economy under incomplete information then draw a tight link between the existence of an aggregate representation and the irrelevance of incomplete information.

In our first proposition, we show that when the elasticity of final goods aggregation is unity, incomplete information has no consequence for either aggregate or sectoral quantities in the economy. In this case, irrelevance at the sectoral level can be recovered regardless of the sectoral structure of the economy. The proof of irrelevance proceeds by showing that *local* prices are in fact sufficient statistics for the aggregate state of the economy, and that information on this state is all that firms need to forecast the marginal value of additional investment. In short, local sectoral prices have a remarkable ability to transmit

and the current one, in which there is a finite set of agent types with different information, we will call the latter "diverse information."

[5]See Woodford (2002) for an illuminating discussion and Lorenzoni (2009) or Melosi (2014) for more recent examples.

the information relevant for investment choices, even when those choices depend on all shocks hitting the economy.

This proposition is related to the findings of Hellwig and Venkateswaran (2014), who also describe circumstances in which market-based information leads to irrelevance of incomplete information. In their environment, irrelevance arises from the static optimality conditions of a price-setting firm and deviations occur when firms face dynamic considerations or strategic complementarities in their price-setting decisions. In contrast, our result emerges from the full general equilibrium relations of the economy and holds even though the firm's investment choice is dynamic and strategically related to that of other sectors.

In our second proposition, we show that an appropriately symmetric version of the economy can deliver aggregate dynamics that are identical to the full-information economy even when sector-level dynamics are not. The key requirement for aggregate irrelevance, beyond symmetry, is that agents observe a variable that reveals the aggregate state of the economy. With this information, firms can use the structure of the economy, including market-clearing conditions, to back out the average actions of other sectors. In general, this aggregate information is not sufficient to determine the optimal investment choice of that particular sector. Nevertheless, if any one sector under-invests relative to the full-information benchmark, the symmetry of the information structure implies that other sectors will over-invest by an offsetting amount. Thus, *mistakes always cancel out*.

In our final proposition, we consider a similarly symmetric economy in which firms must distinguish between local and aggregate disturbances to productivity. In general, firms can no longer perfectly infer the realization of shocks and therefore cannot reproduce the full information equilibrium. Nevertheless, we show that beliefs about the aggregate economy remain common knowledge and, therefore, aggregate quantities remain isomorphic to those of a single-sector economy in which the representative firm forecasts future aggregate productivity based on its observations of current average productivity. Thus, even when the economy is complex enough that agents cannot infer the true shocks hitting the economy, the role for dispersion of information, and the associated hump-shaped dynamics, disappears with market-based information.

After establishing these analytical results, we extend the analysis to a quantitatively realistic version of the multi-sector model. Our numerical simulations show that, if firms make investment choices based on an exogenous information structure consisting of their own productivity and a noisy local signal about average productivity, the economy indeed delivers realistic gradual hump-shaped responses of aggregate investment to productivity shocks. We then show that, once firms are free to condition their investment choices on the information embedded in their local markets, the irrelevance results of the earlier sections reemerge very robustly. Yet, sector-level responses are generally different and individual sectors are not able to determine the sectoral distribution of shocks.

Finally, we calibrate our model to match the empirical input-output structure of the United States economy, and solve the model using processes for aggregate and sectoral TFP estimated to match Jorgenson et al. (2013)'s sectoral data. This version of the model violates the analytical conditions shown to imply aggregate irrelevance and it delivers substantial heterogeneity of expectations about which shocks are hitting the economy. Nevertheless, aggregate dynamics remain remarkably similar to those of the corresponding full-information model. We conclude our quantitative analysis by demonstrating that, despite the complexity of intersectoral linkages, market-based information almost perfectly reveals firms' optimal sectoral investment choices and the aggregate state of the economy.

The paper proceeds as follows. In Section 2, we motivate our results using a simple reduced-form model of intersectoral linkages. In Section 3, we describe the full model and, in Section 4, we establish a set of analytical results characterizing the cases in which information is irrelevant for either sectoral or aggregate outcomes. Section 5 performs a series of numerical experiments to demonstrate the importance of deviations from the assumptions underlying the analytical results. Section 6 calibrates the input-output structure and the exogenous processes of the economy to match US data, and examines the consequences of incomplete information in an empirically realistic setting. Section 7 concludes.

2 Motivating Example

Before presenting our baseline dynamic economy, we examine a simple reduced-form environment that captures the key intuitions for our irrelevance results. Sectors in the economy are indexed by $i \in \{1, 2, ..., N\}$, and each sector is populated by a representative agent who chooses the optimal action

$$q_i = \theta_i + E^i \left[\sum_{j=1}^{N} \alpha_{ij} q_j \right]. \tag{1}$$

According to (1), the optimal action for agents in sector i depends on their own private fundamental, $\theta_i \sim N(0, 1)$, and the actions of other sectors in the economy according to the "linkages" $\{\alpha_{ij}\}$. This structure emulates the interdependence of firms' choices in a general equilibrium input-output model. Though the reaction function in (1) makes no explicit reference to prices, the $\{\alpha_{ij}\}$ arise in equilibrium in our microfounded model because the actions of other firms influence the price of sector i's inputs and output.

Agents form their expectations through their observations of their own fundamental, θ_i, and a subset of endogenous signals, p_i, determined according to

$$p_i = \bar{q} - q_i, \tag{2}$$

where $\bar{q} \equiv \frac{1}{N}\sum_{i=1}^{N} q_i$. The relationship in (2) corresponds to the determination of relative prices in our fully-fledged model, since goods that are scarce relative to average have higher prices.

For our first result, we assume that agents i's observations are given by

$$\Omega_1^i \equiv \{p_i, \theta_i, \{p_j, \forall j \text{ s.t. } \alpha_{ij} > 0\}\}.$$

In words, agents in each sector observe their own productivity, their own output price, and the prices of the firms with whom they interact directly. This information assumption corresponds to the "market-consistent" information assumption we maintain in our dynamic economy.

For our first result, we show that if agents access the observation Ω_1^i, then equilibrium of the model is the same as under full information. To see this, notice that agents' observation of their own relative price, p_i, and their own action, q_i, can be used to infer $\bar{q}$ via (2). This aggregate information can then be combined with the price p_j to infer the equilibrium action q_j for any sector j for which $\alpha_{ij} > 0$. Since the sector's fundamental θ_i and the actions of firms for which $\alpha_{ij} > 0$ are sufficient statistics for the optimal action of agent i, they will always take this action in equilibrium. This result turns out to be completely analogous to our Proposition 1.

With some restrictions on the structure of sectoral linkages, aggregate outcomes may correspond to full information with even less information than contained in Ω_1^i. For our second result, we assume that the economy is circular, i.e. that $\alpha_{ij} = \alpha$ when $j = i + 1$ and $\alpha_{ij} = 0$ for all $j \neq i + 1$.[6] We assume agents' observations are given by

$$\Omega_2^i \equiv \{p_i, \theta_i\}.$$

If agents also had access to p_{i+1}, this would be a special case of the first result and only the full information equilibrium could obtain.

It turns out that, even though individual sectors make mistakes, the aggregate economy behaves just as it does under full information. The proof of this result is slightly more involved. Recall that, since sector i knows its own action, observing p_i is equivalent to observing $\bar{q}$. Hence, in any symmetric equilibrium, the agents in sector i will follow a policy of the form

$$q_i = \omega_1 \theta_i + \omega_q \bar{q}$$

where the coefficients ω_1 and ω_q are to be determined. Averaging this expression across q_i and rearranging yields an expression, $\bar{q} = (1 - \omega_q)^{-1}\omega_1\bar{\theta}$, describing $\bar{q}$ a function of the

[6] In this notation, indexes greater than N "circle back" so that e.g. the index j = N+1=1.

average fundamental, $\bar{\theta}$.

Already we can draw two important conclusions: (1) the aggregate action depends only the average fundamental in the economy, as it does under full information and (2) agent i's information reveals the aggregate state, $\bar{\theta}$. Since $\bar{\theta}$ is known, agent i needs only to forecast θ_{i+1} to determine her optimal action.

Agents in sector i must infer θ_{i+1} using their (implicit) observation of aggregate conditions, so their inference is bound to be imperfect. Since fundamentals are symmetrically distributed and θ_i is observed, we have

$$E[\theta_i|\Omega_2^i] = \frac{N}{N-1}\left(\bar{\theta} - \frac{1}{N}\theta_i\right),$$

which can be used to evaluate[7]

$$\bar{q} = (1-\omega_q)^{-1}\omega_1\bar{\theta} = (1-\alpha)^{-1}\bar{\theta}.$$

This is exactly the value of $\bar{q}$ under full information. Hence, even though agents make mistakes in each sector, those mistakes cancel out on average across the sectors. This result is exactly analogous to our Proposition 2.

In the sections that follow, we show that these results are not special. Instead, similar results emerge in a micro-founded, dynamic business cycle model with sectoral interlinkages. Moreover, even when the conditions for exact information irrelevance are not met, reasonable calibrations of the full economy imply almost no effect of incomplete information so long as firms have access to market-generated information.

3 Multi-Sector Model

We consider a discrete-time, island economy in the vein of Lucas (1972). The economy consists of a finite number of islands, each corresponding to a sector of the economy. On each island/sector resides a continuum of identical consumers and identical locally owned firms, all of whom are price-takers. Consumers derive utility from consumption and experience disutility from supplying labor. The output of firms in each sector is supplied either as an

[7]Plugging this expectation and the conjectured policy into the best response function yields

$$\begin{aligned} q_i &= \theta_i + \alpha E[q_{i+1}|\Omega_2^i] \\ &= \theta_i + \alpha\left(\omega_1 E[\theta_i|\Omega_2^i] + \omega_q\bar{q}\right) \\ &= \underbrace{\left(1-\omega_1\frac{\alpha}{N-1}\right)}_{=\omega_1}\theta_i + \alpha\underbrace{\left(\frac{N}{N-1}(1-\omega_q)+\omega_q\right)}_{=\omega_q}\bar{\theta}. \end{aligned}$$

Solving the fixed point equations above gives $\omega_1 = \frac{N-1}{N-1+\alpha}$ and $\omega_q = \frac{N\alpha}{N-1+\alpha}$.

intermediate input for other sectors or as an input for a single final-good sector, exactly as in Long and Plosser (1983) and subsequent literature. The final good sector does not employ any labor or capital, and its output is usable both as consumption and as capital in the production of intermediates. Since the price of the aggregate final good is observed by all islands, it is common knowledge and we treat it as a numeraire.

3.1 Households

The representative household on island $i \in \{1, 2, ...N\}$ orders sequences of consumption and labor according to the per-period utility function, $u(C, L)$. Household income consists of wages paid to its labor and the dividend payouts of the firms on its island. Workers move freely across firms within their island but cannot work on other islands. Thus, the budget constraint of household on island i in period t is given by

$$C_{i,t} \leq W_{i,t} L_{i,t} + D_{i,t}, \tag{3}$$

where $C_{i,t}$ and $L_{i,t}$ are island-specific consumption and labor, respectively, for time t, and $W_{i,t}$ and $D_{i,t}$ are the island-/sector-specific wage and dividend paid by firms for time t, denominated in terms of the final (numeraire) good.

The household maximizes

$$\max_{\{C_{i,t}, L_{i,t}\}_{t=0}^{\infty}} E_t^i \sum_{t=0}^{\infty} \beta^t u\left(C_{i,t}, L_{i,t}\right), \tag{4}$$

subject to the budget constraint in (3). The expectation operator $E_t^i[V]$ denotes the expectation of a variable V conditional on the information set, Ω_t^i, for island i at time t. The first-order (necessary) conditions for the representative consumer's problem are

$$\begin{aligned} u_{c,t}\left(C_{i,t}, L_{i,t}\right) &= \lambda_{i,t}, & (5)\\ -u_{l,t}\left(C_{i,t}, L_{i,t}\right) &= \lambda_{i,t} E_t^i[W_{i,t}], & (6) \end{aligned}$$

where $\lambda_{i,t}$ is the (current-value) Lagrange multiplier for the household's budget constraint for period t. Under the assumption of market-consistent information, which we describe presently and maintain throughout this paper, consumers will observe both the aggregate price and their wage, so that the first-order condition (6) always holds *ex post* (i.e., without the expectation operators.)

3.2 Production Sector

Output in each sector $i \in \{1, 2, ..., N\}$ is produced according to the production function

$$Q_{i,t} = \Theta_{i,t} F\left(K_{i,t}, L_{i,t}, \{X_{ij,t}\}; \{a_{ij}\}\right), \tag{7}$$

where $\Theta_{i,t}$ is the total factor productivity of the representative firm on island i, $K_{i,t}$ and $L_{i,t}$ are the amounts of capital and labor used, and $X_{ij,t}$ denotes the quantity of intermediate good j used by the firms in sector i. The time-invariant parameters $\{a_{ij}\}$ describe the technology with which goods are transformed into output in sector i. We will use the convention that $a_{ij} = 0$ whenever good j is irrelevant to sector i's production. We summarize the input-output structure of the economy with the $N \times N$ matrix, IO, whose (i, j)'th entry is α_{ij}, where α_{ij} denotes the share of good j in sector i's output. Note that $\alpha_{ij} = 0$ whenever $a_{ij} = 0$ and vice versa.

Firms in each sector i take prices as given and choose all inputs, including the next period's capital stock, so as to maximize the consumers' expected present discounted value of dividends, where expectations are with respect to the island-specific information set. We assume a standard capital accumulation relation

$$K_{i,t+1} = I_{i,t} + (1 - \delta)K_{i,t}, \tag{8}$$

where $I_{i,t}$ is the investment by the representative firm in sector i. Firm i's profit maximization problem is therefore

$$\max_{\left\{L_{i,t}, \{X_{ij,t}\}_{j=1}^{N}, I_{i,t}, K_{i,t+1}\right\}_{t=0}^{\infty}} E_t^i \sum_{t=0}^{\infty} \beta^t \lambda_{i,t} \left(P_{i,t} Q_{i,t} - W_{i,t} L_{i,t} - \sum_{j=1}^{N} P_{j,t} X_{ij,t} - I_{i,t} \right), \tag{9}$$

subject to equations (7) and (8).[8] Here, $P_{i,t}$ denotes the (relative) price of goods produced in sector i.

We assume that firms always observe the current period price of their inputs and output, an assumption we discuss below. Thus, the firm sets the marginal value product of labor and the relevant intermediate inputs equal to their price, yielding the following intratemporal

[8] Our assumption that firms, rather than consumers, choose future capital contrasts with typical practice in the RBC literature. This assumption is for expositional reasons only. In our baseline model, firms on each island have the same information as consumers and therefore make capital accumulation decisions that are optimal from the consumers' perspective as well.

optimality conditions:

$$W_{i,t} = P_{i,t} \frac{\partial Q_{i,t}}{\partial L_{i,t}}, \tag{10}$$

$$P_{j,t} = P_{i,t} \frac{\partial Q_{i,t}}{\partial X_{ij,t}}, \qquad \forall j \text{ s.t. } a_{ij} > 0. \tag{11}$$

Finally, firm i's first-order conditions with respect to investment and future capital combine to yield

$$P_t = \beta E_t^i \left[\frac{\lambda_{i,t+1}}{\lambda_{i,t}} \left(P_{i,t+1} \frac{\partial Q_{i,t+1}}{\partial K_{i,t+1}} + P_{t+1}(1-\delta) \right) \right], \tag{12}$$

where again $P_t = P_{t+1} = 1$ denotes the price of the aggregate good used for investment.

3.2.1 Final-Good Sector

Competitive firms in the final-good sector aggregate intermediate goods using a standard CES technology,

$$Y_t = \left\{ \sum_{i=1}^{N} a_i^{\frac{1}{\zeta}} Z_{i,t}^{1-\frac{1}{\zeta}} \right\}^{\frac{1}{1-1/\zeta}}, \tag{13}$$

where $\sum_{i=1}^{N} a_i = 1$ and Y_t is the output of the final good, $Z_{i,t}$ is the usage of inputs from industry i, and $\{a_i\}_{i=1}^{N}$ represent exogenous, time-invariant weights in the CES aggregator. Input demands are given by

$$Z_{i,t} = a_i \left(\frac{P_{i,t}}{P_t} \right)^{-\zeta} Y_t. \tag{14}$$

3.2.2 Shocks

We assume the process for $\Theta_{i,t}$ has a factor structure, consisting of a common component, A_t, and a sectoral component, $\varsigma_{i,t}$. For any variable V_t, define its log-deviation $\hat{v}_t \equiv \log(V_t/V)$. We assume that log productivity, $\hat{\theta}_{i,t}$, evolves according to

$$\hat{\theta}_{i,t+1} = \mu_i \hat{a}_t + \hat{\varsigma}_{i,t+1} \tag{15}$$

$$\hat{a}_{t+1} = \rho^A \hat{a}_t + \sigma_A \epsilon_{t+1} \tag{16}$$

$$\hat{\varsigma}_{i,t+1} = \rho_i^{\varsigma} \hat{\varsigma}_{i,t} + \sigma_i \epsilon_{i,t+1}. \tag{17}$$

where the iid shocks ϵ_t and $\epsilon_{i,t}$ are normal and have unit variance.

3.3 Equilibrium

The equilibrium of the economy is described by equations (5) through (14), the exogenous processes for $\Theta_{i,t}$, and the island-specific market-clearing conditions and resource constraints,

$$Q_{i,t} = Z_{i,t} + \sum_{j=1}^{N} X_{ji,t}, \tag{18}$$

$$P_{i,t}Q_{i,t} = C_{i,t} + I_{i,t} + \sum_{j=1}^{N} P_{j,t}X_{ij,t}. \tag{19}$$

By Walras' law, we have ignored the aggregate market-clearing condition $Y_t = \sum_{i=1}^{N} C_{i,t} + \sum_{i=1}^{N} I_{i,t}$. Thus, we have 1+9N+N^2 equations in the same number of unknowns: Y_t, $\{P_{i,t}\}_{i=1}^{N}$, $\{W_{i,t}\}_{i=1}^{N}$, $\{C_{i,t}\}_{i=1}^{N}$, $\{\lambda_{i,t}\}_{i=1}^{N}$, $\{Q_{i,t}\}_{i=1}^{N}$,$\{Z_{i,t}\}_{i=1}^{N}$, $\{L_{i,t}\}_{i=1}^{N}$, $\{I_{i,t}\}_{i=1}^{N}$, $\{K_{i,t}\}_{i=1}^{N}$, and $\{X_{ij,t}\}_{i,j=1}^{N}$. Depending on the number of the zeros in the input-output matrix, some of the unknown $X_{ij,t}$ and corresponding first-order conditions in equation (11) will drop out, reducing the size of the system.

3.4 Information

In this paper, we follow the suggestion of Nimark (2008) and Graham and Wright (2010) that, when assessing the role of information frictions, agents should be allowed to learn about the economy based on "market-consistent" information. That is, a firm's information set should include, as a minimal requirement, those prices that are generated by the markets in which it trades. In our context, this means that firms observe and learn from the prices of their output and all inputs with a positive share in their production. In addition to these prices, we also assume that firms observe their own productivity, since it could easily be inferred from any endogenous island-specific variable such as firm profits or the local wage. The following definition makes this assumption precise:

Definition 1. *The* ***market-consistent*** *information set of agents in sector i, denoted by $\Omega_t^{i,MC}$, is given by full histories*

$$\{\Theta_{i,t-h}, P_{i,t-h}, P_{j,t-h}, \forall j \ \ s.t. \ \ \alpha_{ij} > 0\}_{h=0}^{\infty}. \tag{20}$$

For the log-linear approximation to the model considered later, $\hat{\Omega}_t^{i,MC}$ is defined analogously to contain log-level deviations of the same variables.

The macroeconomic literature on intersectoral linkages has traditionally focused on how intersectoral linkages affects the economy-wide propagation of sectoral shocks. Our key observation is that, under the assumption of market-consistent information, the nature of intersectoral trade will be a crucial determinant of the information available to the firms. While the existence of a relatively sparse input-output structure (which is the empirically relevant case) implies that firms directly observe a very small portion of the overall economy, the informational consequences of these observations prove to be disproportionately large.

Notice that we have not allowed firm ownership to trade across islands, thereby preventing households from learning via the corresponding equity prices. This assumption clearly favors a relatively large effect of incomplete information and, in this sense, is a conservative assumption given our finding that information effects are quite small under market-based learning. We have also experimented with allowing sector islands to trade risk free bonds and have found it too limits the scope for information frictions to matter.

Assumptions about information are susceptible to the "Lucas critique," because what agents choose to learn about may be influenced by policy and other non-informational features of the economic environment. The assumption of market-consistent information represents a compromise between assuming an exogenous fixed-information structure and the assumption that agents endogenously design an optimal signaling mechanism according to a constraint or cost on information processing (as suggested by the literature on rational inattention initiated by Sims, 2003.) Because agents form expectations based on prices, the information content of which depends on agents' actions, there is scope for an endogenous response of information to the fundamental parameters governing the environment. Thus, the assumption of market-consistent information offers at least a partial response to the critique: if agents face a discretely lower marginal cost of learning from variables that they must in any case observe in their market transactions, then comparative statics for small changes in parameters may be valid.

4 Irrelevance Results

In this section, we develop several propositions that together provide important benchmarks for when information frictions cannot matter for the dynamics of the model. The propositions in this section follow the tradition in the RBC and information-friction literature and focus on a log-linear approximation of the economy. The first lemma describes cases in which the sectoral economy is isomorphic to a corresponding single-sector economy. The following propositions establish conditions under which incomplete information either (1) affects neither sectoral nor aggregate outcomes, (2) potentially affects sectoral outcomes but has no effect on aggregate outcomes, or (3) affects both aggregate and sectoral outcomes, but with no aggregate consequences of dispersed information.

For the theoretical results in this section, we make several simplifying assumptions. In particular, we assume that intermediate production is Cobb-Douglas in all inputs, sectoral weights in final-good production are symmetric, capital depreciates fully each period, labor is supplied inelastically, and preferences take a CRRA-form with an elasticity of intertemporal substitution equal to τ. The linearization of the model and Cobb-Douglas production are important for our analytical results. However, full depreciation, inelastic labor supply, and CRRA utility only simplify derivation of the results, as we verify numerically in section 5. Finally, for expositional simplicity, we assume in this section that realizations of all shocks are revealed to agents with a one-period lag.[9]

The linearized first-order condition of the consumer in sector i is

$$\hat{c}_{i,t} = -\tau \hat{\lambda}_{i,t}. \tag{21}$$

Intermediate production is characterized by the linearized production function

$$\hat{q}_{i,t} = \hat{\theta}_{i,t} + \alpha_{ik}\hat{k}_{i,t} + \sum_{j=1}^{N} \alpha_{ij}\hat{x}_{ij,t}, \tag{22}$$

where, as noted earlier, the parameters α_{ik} denote the capital share of output in sector i and α_{ij} the share of good j in the output of sector i. We assume that $\alpha_{ik}+\sum_{j=1}^{N}\alpha_{ij} = 1-\phi_l < 1$, so that the share of inelastically supplied labor is positive (or, equivalently, that the economy exhibits decreasing returns to scale.) Recall our convention that $\hat{x}_{ij,t} = 0$ whenever $\alpha_{ij} = 0$.

The firm's optimal choice of input $\hat{x}_{ij,t}$ is given by

$$\hat{p}_{j,t} = \hat{p}_{i,t} + \hat{q}_{i,t} - \hat{x}_{ij,t}. \tag{23}$$

Linearizing the intertemporal optimality condition of the firm, and using the consumer's first-order condition to substitute out $\hat{\lambda}_{i,t}$ yields

$$-\frac{1}{\tau}E_t^i\left[\hat{c}_{i,t} - \hat{c}_{i,t+1}\right] = E_t^i\left[\hat{p}_{i,t+1} + \hat{q}_{i,t+1} - \hat{k}_{i,t+1}\right]. \tag{24}$$

Final-good aggregation with symmetric weights implies that

$$\hat{y}_t = \frac{1}{N}\sum_{i=1}^{N}\hat{z}_{i,t}, \tag{25}$$

with $\hat{z}_{i,t}$ demanded according to

$$\hat{z}_{i,t} = \hat{y}_t - \zeta\hat{p}_{i,t}. \tag{26}$$

[9] Some authors, including Graham and Wright (2010) and Rondina and Walker (2017), have emphasized cases where similar "truncations" would exclude otherwise valid equilibria in which endogenous observables have non-invertible representations. With some minor caveats, the theorems in this section hold without assuming revelation at any horizon. Our numerical experiments suggest that this type of multiplicity does not arise in our model.

Sectoral market clearing implies

$$\hat{q}_{i,t} = s_{iz}\hat{z}_{i,t} + (1 - s_{iz})\sum_{j=1}^{N}\eta_{ji}\hat{x}_{ji,t}, \tag{27}$$

$$\hat{p}_{i,t} + \hat{q}_{i,t} = s_{ic}\hat{c}_{i,t} + s_{ik}\hat{k}_{i,t+1} + (1 - s_{ic} - s_{ik})\sum_{j=1}^{N}\omega_{ij}(\hat{p}_{j,t} + \hat{x}_{ij,t}), \tag{28}$$

where s_{iz} is the steady-state share of sector i output devoted to final-good production, η_{ji} is the fraction of sector i good's usage as an intermediate devoted to sector j, s_{ic} and s_{ik} are the shares of gross value of sectoral output dedicated to consumption and investment, respectively, and ω_{ij} is the share of sector i's payments for intermediates going to sector j.

Equations (22) through (28), plus the exogenous process for $\hat{\theta}_{i,t}$, fully characterize the linearized model. We assume throughout this section that the model is parameterized so that it has a unique stationary equilibrium under full information.

4.1 Aggregate Representations

Under certain circumstances, the equilibrium conditions of the full-information economy can be reduced to a set of aggregate relations that are isomorphic to a single-sector RBC model without intermediate production. Lemma 1 derives a sufficient condition for such an "aggregate representation," and generalizes substantially the result of Dupor (1999). In the subsequent sections, we use this result to derive some general implications about the economy with incomplete, but market-consistent, information.

To find sufficient conditions for the existence of an aggregate representation, we first derive a matrix-representation of the sectoral economy's equilibrium conditions. To begin, observe that using the firm's decision rule for inputs, (23), we can derive an expression for the $\hat{x}_{ij,t}$,

$$\hat{x}_{ij,t} = \hat{p}_{i,t} + \hat{q}_{i,t} - \hat{p}_{j,t}. \tag{29}$$

In the appendix, we derive the equilibrium conditions of the economy as a set of matrix relations. Letting bold-face type represent the vector $\mathbf{x}_t \equiv [\hat{x}_{1,t}, \hat{x}_{2,t}, ..., \hat{x}_{N,t}]'$ for any variable

$\hat{x}_{i,t}$, the equilibrium is described by

$$\mathbf{q}_t = (I-\Phi_x)^{-1}\boldsymbol{\theta}_t + (I-\Phi_x)^{-1}(\Phi_x - IO)\mathbf{p}_t + (I-\Phi_x)^{-1}\Phi_k\mathbf{k}_t \tag{30}$$

$$\mathbf{p}_t + \mathbf{q}_t = \Phi_z(I-IO')^{-1}(\mathbf{z}_t+\mathbf{p}_t) \tag{31}$$

$$\mathbf{z}_t = A_v\mathbf{z}_t - \zeta\mathbf{p}_t \tag{32}$$

$$\mathbf{p}_t + \mathbf{q}_t = \phi_c\mathbf{c}_t + (1-\phi_c)\mathbf{k}_{t+1} \tag{33}$$

$$\mathbf{k}_{t+1} = E_t^f[\frac{1}{\tau}(\mathbf{c}_t - \mathbf{c}_{t+1}) + \mathbf{p}_{t+1} + \mathbf{q}_{t+1}] \tag{34}$$

Equation (30) captures sectoral production functions, after substituting out intermediate demand via equation (29). Here, Φ_k is a diagonal matrix with entries α_{ik}, and Φ_x is a diagonal matrix whose entries are the row-sums of the input-output matrix, $\alpha_{ix} \equiv \sum_{j=1}^N \alpha_{ij}$.

Equation (31) is the market clearing condition for sectoral output, after substituting out intermediate usages, and Φ_z is a diagonal matrix containing the steady-state ratios $\{s_{iz} = Z_i/Q_i\}_{i=1}^N$.

Equation (32) captures demand for sectoral output from the final goods sector. Here, A_v is defined as $1/N$ times an $N \times N$ unit matrix that replicates the column averages of any conformable matrix that it premultiplies. For future reference, let a_v be the first row of A_v, i.e., the row vector that averages the columns of any matrix it pre-multiplies.

Equation (33) captures the island resource constraint for each sector. Here, ϕ_c represents the share of consumption in value-added production, which is common across sectors.

Finally, (34) describes the optimal capital choice of each sector, with $E_t^f[\cdot]$ denoting the full information expectation.

Lemma 1. *Suppose that $\rho^A = \rho_i^\varsigma = \rho$. Then, there exists a vector of weights, w, and a scalar, $\tilde{\alpha}$, such that whenever*

$$(1-\zeta)w\Phi_z(I-IO')^{-1}\mathbf{p}_t = 0, \qquad \forall t, \tag{35}$$

the full-information economy has a (single-sector) aggregate representation given by

$$\tilde{y}_t = \tilde{\theta}_t + \tilde{\alpha}\tilde{k}_t, \tag{36}$$

$$\tilde{y}_t = \phi_c\tilde{c}_t + (1-\phi_c)\tilde{k}_{t+1}, \tag{37}$$

$$\tilde{k}_{t+1} = E_t\left[\frac{1}{\tau}(\tilde{c}_t - \tilde{c}_{t+1}) + \tilde{y}_{t+1}\right], \tag{38}$$

$$\tilde{\theta}_{t+1} = \rho\tilde{\theta}_t + \sigma\tilde{\epsilon}_{t+1}, \tag{39}$$

where $\tilde{y}_t \equiv a_v\mathbf{z}_t$, $\tilde{c}_t \equiv w\mathbf{c}_t$, $\tilde{k}_t \equiv w\mathbf{k}_t$, $\tilde{\theta}_t \equiv a_vH^{-1}\boldsymbol{\theta}_t$, and $H \equiv (1-\zeta^{-1})(I-\Phi_x)\Psi + \zeta^{-1}(I-IO)$.

Proof. See Appendix B. □

In the appendix, we show that the matrix representation of the intertemporal Euler equation (34) simplifies to

$$\mathbf{k}_{t+1} = E_t^f \left[\frac{1}{\tau}(\mathbf{c}_t - \mathbf{c}_{t+1}) + A_v \mathbf{z}_{t+1} - (\zeta - 1)\mathbf{p}_{t+1} \right], \tag{40}$$

implying that the sectoral investment/consumption choice depends only on the aggregate term, $A_v \mathbf{z}_{t+1}$, and the dynamics of future sector-specific prices. When the condition in (35) is satisfied, the effect of the last term aggregates to zero, so that aggregate dynamics can be determined independent of sectoral allocations. When such a representation exists, we refer to $\{\tilde{\theta}_t, \tilde{k}_t\}$ as the *notional aggregate state* of the economy. Since equilibrium dynamics can be alternatively represented with an infinite moving average process in $\tilde{\theta}_t$, the notional aggregate state can also be summarized by the history $\{\tilde{\theta}_{t-h}\}_{h=0}^{\infty}$.

Equation (35) can be satisfied in several ways to yield an aggregate representation. The most direct path is given by Corollary 1:

Corollary 1. *The economy has an aggregate representation whenever $\zeta = 1$.*

Proof. The result follows from observing that the left-hand side of (35) is premultiplied by $\zeta - 1$. □

The corollary states that whenever the final good aggregator is Cobb-Douglas, as it typically is in the related literature, the full-information economy has an aggregate representation *regardless* of the input-output structure. Intuitively, this result is a consequence of the constant-share implication of Cobb-Douglas production. Since each intermediate good has a constant share in each of the other sector's production, the real value of output in each sector moves in tandem. For same reason, the returns to holding capital depend only on my own sector's capital stock and expected aggregate conditions, as indicated by equation (40) when ζ=1. Hence, both the static and dynamic decisions of each sector are driven only by aggregate conditions.

Corollary 2 next shows that, even when $\zeta \neq 1$, the linearized economy may still have a representation of aggregate quantities that does not make reference to sector-specific variables. This happens when the economy is symmetric in the sense defined by Dupor (1999).

Definition 2. *The input-output matrix, IO, is* ***circulant*** *if its rows consist of the elements*

$\alpha_1, \alpha_2, ..., \alpha_N$ *and can be rearranged to take the following form*

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_N \\ a_N & \alpha_1 & \dots & \alpha_{N-1} \\ \vdots & & \ddots & \vdots \\ \alpha_2 & \alpha_3 & \dots & \alpha_1 \end{pmatrix}. \tag{41}$$

Corollary 2. *If the input-output matrix of the economy is circulant, then the full-information economy has a (single-sector) aggregate representation that is identical to a representative agent economy whose equilibrium conditions are given by*

$$\hat{y}_t = (1-\alpha_x)^{-1}\hat{\theta}_t + (1-\alpha_x)^{-1}\alpha_k\hat{k}_t, \tag{42}$$

$$\hat{y}_t = (1-\alpha_x)^{-1}\alpha_l\hat{c}_t + (1-\alpha_x)^{-1}\alpha_k\hat{k}_t, \tag{43}$$

$$\hat{k}_{t+1} = E_t^f\left[\frac{1}{\tau}E_t^f\left(\hat{c}_t - \hat{c}_{t+1}\right) + \hat{y}_{t+1}\right], \tag{44}$$

$$\hat{\theta}_{t+1} = \rho\hat{\theta}_t + \hat{\epsilon}_{t+1}, \tag{45}$$

where α_x is the row sum of the IO matrix, $\alpha_k = 1-\alpha_l-\alpha_x$ is the capital share of the sectoral economy, and each aggregate variable $\hat{v}_t \equiv \frac{1}{N}\sum_{i=1}^{N}\hat{v}_{i,t}$, with $\hat{v}_{i,t}$ being the corresponding sectoral variable.

Proof. See Appendix B. □

Unlike the more general case, in the circulant economy the notional aggregate is simply an equally-weighted average of sectoral productivities. Importantly, the argument in Appendix B relies on the market-clearing condition that log-relative prices in the economy sum to zero, a feature that is not directly exploited in establishing the result of Corollary 1. This special feature of the circulant economy—namely, that the notional aggregate state weights sectoral quantities in the same way as the aggregate price index weights sectoral prices—gives rise to the general equilibrium canceling of prices, a result that is also key to our proof of aggregate irrelevance in Section 4.3.

Before proceeding to the irrelevance propositions, it is convenient to define the concepts of *action-informative* and *aggregate-informative* information sets.

Definition 3. *An information set $\hat{\Omega}^i$ is **action informative** for agents of type i if, in the full-information economy, it is a sufficient statistic for type i's optimal action.*

Definition 4. *An information set $\hat{\Omega}^i$ is **aggregate informative** if, in the full-information economy, it is a sufficient statistic for the notional aggregate state.*

Notice that, generally, information can be action informative without being aggregate informative and vice-versa. Whenever all agents in an economy have access to an action informative information set, there exists an equilibrium of the economy that is identical to that of the full-information economy. To see that this must be the case, consider the choice of an individual with an action informative information set when all other agents in the economy behave according to the prescriptions of the full-information economy. By the definition of action informativeness, the individual's information must reveal her optimal action. By construction, however, they can do no better than to take that action and the same applies to all other agents in the economy; the conjecture that the equilibrium replicates the full information outcome is sustained.

4.2 Sectoral Irrelevance

Proposition 1 shows that, with a restriction on the elasticity of the final-goods aggregator (ζ), market consistent information is sufficient to reproduce the full-information equilibrium *regardless of the input-output structure.* The additional restriction on the intertemporal elasticity (τ) below simplifies the proof, but extensive numerical exploration suggests to us that it is not necessary for the result.

Proposition 1. *Suppose that $\tau = \infty$, $\zeta = 1$ and $\rho^A = \rho_i^\varsigma = \rho$. Then (i) market consistent information, $\hat{\Omega}_t^{i,MC}$, is both aggregate and action informative and (ii) the equilibrium of the diverse-information model with information sets $\hat{\Omega}_t^{i,MC}$ is identical to the that of the full information economy.*

Proof. See Appendix B. □

Proposition 1 highlights a striking coincidence in the economy with Cobb-Douglas aggregation: market-consistent information is simultaneously action informative and aggregate informative and optimality never requires firms to forecast outcomes at the sectoral level. To arrive at this conclusion, we show in the appendix that, under full information, sectoral capital accumulation in each sector is given by

$$k_{i,t+1} = c\tilde{\theta}_t, \tag{46}$$

for a constant c. That is, the optimal capital accumulation choice of the firm in each sector depends only on the notional aggregate productivity that appears in Lemma 1. This result is again a consequence of the constant share implication of Cobb-Douglas production, which implies that aggregate conditions are a sufficient statistic for the marginal product of capital.

We next show that the market-consistent information set is precisely what is required to infer the required linear combination of sectoral shocks. To do this, we construct a signal $s_{i,t}^p \equiv \hat{p}_{i,t} - \sum \alpha_{ij}\hat{p}_{j,t} - \hat{\theta}_{i,t}$ from the set of market consistent observables in sector i, and show that it is equal to

$$s^p_{i,t} = (1-\alpha_{ik})\tilde{\theta}_t + \gamma \tilde{k}_t \tag{47}$$

for a constant γ. Hence, the signal $s^p_{i,t}$ corresponds to noiseless signal on aggregate productivity[10]: once one controls for direct costs of production, the sectoral price become a sufficient statistic for the aggregate state of the economy.

The proposition provides an important benchmark for assessing the importance of information frictions under the assumption of market consistent information: information transmission of payoff relevant states is complete and does not depend on the sparsity, balance, or degree of linkages. Moreover, inference of aggregate conditions is sufficient for achieving this irrelevance: as long as agents have access to some variable that reveals this state, such as aggregate output, the full-information equilibrium can be supported.

4.3 Aggregate Irrelevance

Proposition 1 establishes aggregate irrelevance from the "bottom-up," by showing the existence of equilibria in which all firms take the same actions as they would under full information. Proposition 2, in contrast, proceeds via a "top-down" logic by showing that equilibrium conditions can impose restrictions on aggregates independently of what they imply for sector-level dynamics. It establishes that in a circulant input-output economy, incomplete information has no aggregate consequences, regardless of the value of ζ, so long as agents have access to an aggregate informative variable.

Proposition 2. *Suppose that $\rho^A = \rho^\varsigma_i = \rho$, $\mu_i = \mu$, that the input-output matrix is circulant, and that*

$$\hat{\Omega}^i_t = \left\{ \hat{\Omega}^{i,MC}_t, \{\hat{\theta}_{t-h}\}^\infty_{h=0} \right\}. \tag{48}$$

Then any symmetric equilibrium of the diverse information economy has the same aggregate dynamics as the full information equilibrium.

Proof. See Appendix B. □

Corollary 3. *Any equilibrium of the model with $\hat{\Omega}^i_t = \left\{ \hat{\Omega}^{i,MC}_t, \{\hat{\theta}_{t-h}\}^\infty_{h=0} \right\}$ is also an equilibrium of the model with $\hat{\Omega}^i_t = \left\{ \hat{\Omega}^{i,MC}_t, \{\hat{v}_{t-h}\}^\infty_{h=0} \right\}$, where $\{\hat{v}_{t-h}\}^\infty_{h=0}$ is aggregate informative.*

This result is notable for two reasons. First, the presence of complementarities in decisions means that higher-order expectations matter for the decisions of individual firms. In the context of price-setting firms, such complementarity typically leads to large aggregate

[10]Since we have assumed full revelation after one period and the aggregate state $\tilde{k}_t$ is predetermined, its presence in the signal is innocuous. Nevertheless, the proof of action informativeness in the appendix does not rely on lagged revelation.

consequences of information frictions, and increased persistence in particular. Second, the result on aggregates holds even though sectoral expectations and choices can be substantially different under market-consistent information. Sectoral mistakes cancel each other out, despite the fact that *no law of large numbers is being invoked*, nor does any apply in our economy.

Technically, the key to the results above is that agents have some means of inferring the aggregate state of productivity from their information set either directly, as in Proposition 2, or indirectly, as in Corollary 3. When they do, agents can track aggregates in the economy independent of their ability to track the idiosyncratic conditions relevant to their choices. Since average expectations must then be consistent with the common-knowledge aggregate dynamics, expectational mistakes—and therefore mistakes in actions—must cancel out, regardless of what happens at the sectoral level.

To see the logic of the result, consider the incomplete-information version of equation (40),

$$\hat{k}_{i,t+1} = E_t^i \left[\frac{1}{\tau}(\hat{c}_{i,t} - \hat{c}_{i,t+1}) + a_v \mathbf{z}_{t+1} - (\zeta - 1) p_{i,t+1}. \right] \tag{49}$$

Given that agents have access to the current aggregate state, the full-information forecast of the aggregate $a_v \mathbf{z}_{t+1}$ is common knowledge. Therefore, if a particular sector i invests more than it does under full information, this error must be due to a deviation in the sector's forecast of its own relative price. But firms cannot all, on average, expect their own relative prices to increase. The symmetry of the input-output structure implies that any price signal that is good news for sector i's future relative price must also be bad news for some sector j's future price. The proof of Proposition 2 shows that the effect of these sector-level errors is perfectly offsetting, so that summing the Euler equation in (49) yields the Euler equation of the aggregate representation of the full-information economy, (44), of Corollary 2.

The proof also makes clear that the inclusion of market-consistent information is not essential to this result; other symmetric information structures that also reveal the notional aggregate state deliver the same aggregate irrelevance. In principle, these results permit very large implications of limited information at the sectoral level while perfectly imitating the aggregate dynamics of the full-information model. Generating examples that demonstrate such a large disconnect is rather easy, as we show in section 5. However, in practice we find that it is hard to do this for realistic calibrations and specifications of the information structure.

Conversely, while the ability to forecast aggregates is essential for the exact results in Proposition 2 and for formulating intuition, in practice the consequences of removing the aggregate-informative variable $\hat{v}_t$ from the market-consistent information set is small. In the next sections, we show that relative prices, in conjunction with the observation of own-sector productivity, do a nearly perfect job of revealing the aggregate state.

4.3.1 Disentangling Temporary and Persistent Productivity

So far in this section, we have assumed that the common and sectoral productivity components have the same persistence. This assumption is important because it implies that average productivity in the economy has an AR(1) representation, and hence that current average productivity is a sufficient statistic for the full information forecast of future average productivity. The same sufficient statistic argument lies behind the common persistence assumption in Proposition 2 of Hellwig and Venkateswaran (2014). Our last proposition relaxes the common persistence assumption and shows that, unlike Hellwig and Venkateswaran (2014), information dispersion still has no affect on aggregate outcomes. Instead, the sectoral model with incomplete information is equivalent to an aggregate economy in which the agents have common but incomplete information about the persistent and transitory components of productivity.

Proposition 3. *Suppose that $\tau = \infty$, $\zeta = 1$, $\mu_i, \rho_i^{\varsigma}$ and σ_i are common across sectors, the input-output matrix is circulant, and the information set of firms is $\hat{\Omega}_t^{i,MC}$. Then the equilibrium of the diverse-information model is equivalent to that of the single-sector economy in which the representative agent observes $\hat{\theta}_t$, but not the decomposition between common and sectoral shocks.*

Proof. See Appendix B. □

As with Proposition 2, symmetry in the economy causes sector-level mistakes to cancel exactly. This case is distinct, however, because the information of agents is not sufficient to infer the decomposition of the current aggregate state $\tilde{\theta}_t$ into its aggregate and idiosyncratic components. When $\rho^A \neq \rho^{\varsigma}$, forecasts are not the same as full information, since knowing whether a change in productivity originates in the common or sectoral components would be helpful for forecasting future average productivity. Nevertheless, without independent information on the source of productivity changes, forecasts remain identical across agents so that disagreement can have no effect on equilibrium aggregates.

5 Beyond Irrelevance

We now examine the degree to which the analytical results derived above apply to the more general model outlined in Section 3. We therefore relax the restrictions of Section 4 and reinstate partial depreciation of capital and labor-leisure choices in the model. Moreover, we work with more general functional forms for the utility and production functions, calibrating the associated parameters to realistic values.

Table 1: Baseline parameterization of the model.

Parameter	Concept	Value
N	Number of sectors	6.00
δ	Capital depreciation	0.05
κ	Capital-labor elasticity	0.99
ξ	Elasticity among intermediates	0.33
σ	Elasticity between composite inputs	0.20
ζ	Final goods elasticity	1.50
ϕ_x	Share of intermediate inputs	0.60
$\tilde{\alpha}$	Capital share of value-added	0.34
β	Discount factor	0.99
τ	Intertemporal elasticity	0.50
φ^{-1}	Implied Frisch elasticity = 1.9	15.00
ρ_ς	AR coeff. sectoral prod. shocks	0.90
ρ_A	AR coeff. agg shock (when used $\rho_\varsigma = 0.7$)	0.95

5.1 Functional Forms and Calibration

For our quantitative analysis, we use the per-period utility function

$$u\left(C,L\right) = \frac{\left(C(1-L)^{\varphi}\right)^{1-\frac{1}{\tau}} - 1}{1-\frac{1}{\tau}}, \tag{50}$$

where τ is again the elasticity of intertemporal substitution, and the Frisch elasticity of labor supply is given by $\frac{1-\bar{L}}{\bar{L}}\frac{1}{1+\varphi(1-\tau)}$, where $\bar{L}$ is the average fraction of overall hours dedicated to production.

On the firm side of the economy, we assume that the production function $F(\cdot)$ takes the form of a nested-CES technology:

$$F\left(K_{i,t}, L_{i,t}, \{X_{ij,t}\}\right) = \left[b_{i1}\left\{\sum_{j=1}^{N} a_{ij} X_{ij,t}^{1-\frac{1}{\xi}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}} + b_{i2}\left\{a_{il}L_{i,t}^{1-\frac{1}{\kappa}} + a_{ik}K_{i,t}^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}}\right]^{\frac{1}{1-1/\sigma}} \tag{51}$$

where ξ is the elasticity of substitution between intermediate inputs, κ is elasticity of substitution between capital and labor, and σ is the elasticity of substitution between the composite intermediate input and the composite capital-labor input. Finally, a_{il}, a_{ik}, b_{i1} and b_{i2} are production parameters that are set to match the (cost) shares of various inputs. Without loss of generality, we normalize $b_{i1} = b_{i2} = 1$.

The calibrated parameters with their associated targets are summarized in Table 1, with additional details on calibrating the production structure provide in Appendix A. For this stylized example, we take the number of sectors to be six. Although this is a relatively small number, none of our qualitative results depend on this choice. A few other parameter choices warrant special attention. First, we calibrate the elasticity between the two composite inputs, $\sigma = 0.20$, well below unity. This value is in line with the estimates discussed in the working-paper version of Moro (2012). We calibrate the share of intermediate inputs to be 0.6, which is the value suggested by Woodford (2003). These two choices are crucial in determining the degree of complementarity in the model, as we discuss in the next section. Additionally, we set the final-good elasticity $\zeta = 1.5$, which is higher than the value used in Horvath (2000) and somewhat less than what is typically assumed in the new-Keynesian literature (which instead focuses on the markups generated by imperfect competition). We take $\varphi = 15$, which implies a Frisch elasticity in our model of slightly under two. Finally, the capital depreciation rate is set to a standard value.

Solving the model poses a technical challenge because agents must "forecast the forecasts of others," as in Townsend (1983), and because they must condition these expectations on the information embodied in endogenous variables. Appendix C summarizes our numerical approach to solving the model.

5.2 Intersectoral Linkages and Complementarities

Before proceeding to our numerical results, it is helpful to understand the sources and strength of the strategic interactions generated by the introduction of an intermediate production structure. In new-Keynesian environments, strategic complementarities pertain to the static price-setting decision of firms.[11] In contrast, here firms' decisions are inherently dynamic, which makes an analytical treatment more difficult. To maintain tractability, we consider the strategic interactions in the steady state of a symmetric two-sector version of the model from Section 4. Specifically, we consider the steady-state investment choice of sector *1*, and examine its response to a percentage deviation, Δ, of sector *2* investment from its steady-state equilibrium value. In Appendix B, we show that the resulting investment choice of sector *1* is given by

$$\hat{k}_1^* = \hat{k}_{1,ss} + \frac{\phi_k}{1+\phi_k}\Delta. \tag{52}$$

The parameter $\phi_k > 0$ is, therefore, the relevant measure of strategic complementarity in the model.

In order to do simple comparative statics for ϕ_k vis-à-vis various parameters of the model, it is helpful to specialize, for the time being, to the Cobb-Douglas production function with

[11] Authors including Basu (1995), Nakamura and Steinsson (2010), and Carvalho and Lee (2011) have examined how intermediate inputs influence complementarities in firms' price-setting decisions.

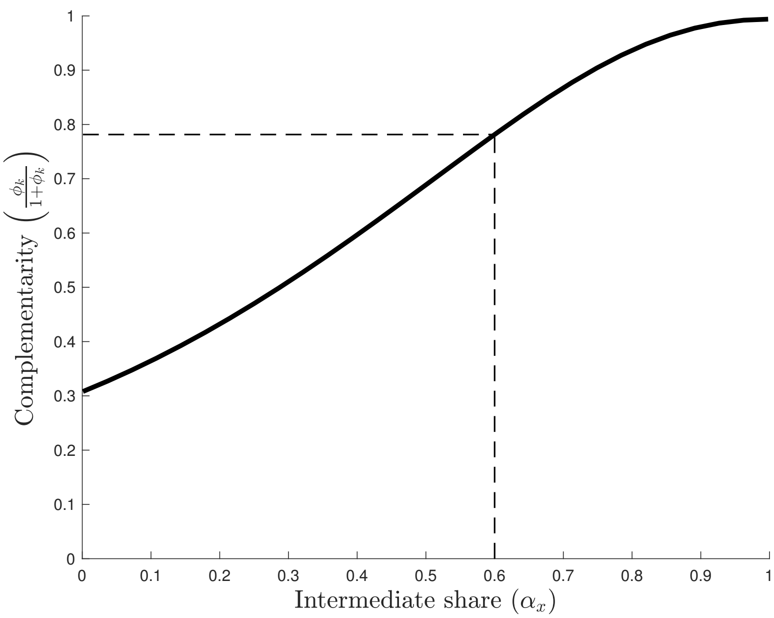


Figure 1: Steady-state complementarities for the general model.

a fixed supply of labor. Specifically, assume that

$$F(K, L, X) = K^{\tilde{\alpha}_k(1-\alpha_x)} L^{(1-\tilde{\alpha}_k)(1-\alpha_x)} X^{\alpha_x}. \tag{53}$$

In this formulation, $\tilde{\alpha}_k = \alpha_k/(1-\alpha_x)$ represents the shares of capital in *value-added* in the economy and α_x is the economy-wide share of intermediates in production. In this special case, we have that

$$\phi_k = \frac{1}{2}\left(\frac{\tilde{\alpha}_k}{1-\tilde{\alpha}_k}\right)\left(\frac{(1+\alpha_x)^2}{(1-\alpha_x)^2\zeta + 4\alpha_x}\right). \tag{54}$$

It follows immediately that the complementarity in decision making is higher when (1) the share of capital in value added ($\tilde{\alpha}_k$) is very large, (2) the share of intermediates (α_x) is large, and (3) the input elasticity (ζ) in the final-goods sector is relatively low. Notice, in particular, the contrast of comparative static (3) relative to standard new-Keynesian environments in which higher elasticities lead to greater, rather than smaller, pricing complementarities.

Since complementarity is increasing in α_x, the limit as $\alpha_x \to 1$ delivers an upper bound on the degree of complementarity:

$$\lim_{\alpha_x \to 1} \phi_k = \frac{1}{2}\left(\frac{\tilde{\alpha}_k}{1-\tilde{\alpha}_k}\right). \tag{55}$$

Thus, under a standard calibration with a capital share of one third, a one-percentage exogenous increase in sector *2*'s capital choice can deliver no more than a $\frac{1/3}{1+1/3} = 0.25$-percentage increase in sector *1*'s own capital choice—a relatively weak complementarity by the standards of the new-Keynesian literature.

Table 2: Relative standard deviations for circle production structure with sectoral shocks only.

	Output	**Cons.**	**Inv.**	**Hours**	**Sect. Inv.**
Full Information	1.00000	0.69781	1.88524	0.35229	1.999461
Market-consistent + GDP	1.00000	0.69781	1.88524	0.35229	1.999506
Market-consistent lagged + GDP	1.00000	0.69781	1.88524	0.35229	2.565297
Market-consistent	1.00001	0.69781	1.88527	0.35230	1.999512
Own-price only	1.03151	0.70732	1.98956	0.38088	2.046435
Exogenous	0.77456	0.64663	1.16956	0.17471	1.262126

In the more general version of the model, the steady-state investment complementarity differs from the value in the fixed-labor, Cobb-Douglas version of the model discussed above. Figure 1 plots the value of $\frac{\phi_k}{1+\phi_k}$ against the share of intermediates under the baseline calibration of the general model. Although the comparative statics derived above are robust, the bound derived for the specialized Cobb-Douglas case turns out to be quite conservative. This difference is driven primarily by the introduction of an endogenous labor choice and our calibration of a much-lower-than-one elasticity of substitution between the intermediate good and the capital-labor composite. Under our baseline calibration of an intermediate share of 0.6, the value of this complementary is roughly $\frac{\phi_k}{1+\phi_k} = 0.78$. Though slightly lower than the standard new-Keynesian calibration (see e.g. Woodford, 2003), this value of complementarily is sufficient to generate a strong role for higher-order expectations in equilibrium dynamics.

5.3 Uncorrelated Shocks

We begin by considering the model with symmetric and independent sectoral shock processes ($\mu_i = 0$ and $\rho_i^{\varsigma} = 0.7$) and a stylized symmetric circle production structure,

$$IO^{cir} = \alpha_x \times \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (56)$$

which is a special case of the circulant production structure discussed earlier. We choose this sectoral structure because it is extremely sparse and thus corresponds to an especially restricted set of observable prices for the market-consistent information set.

Recall that since the version of the model we are considering has a finite number of sectors, sectoral shocks always have aggregate implications. The first row of Table 2 summarizes the aggregate moments of the full-information model. The model does a relatively good job of capturing the relative variances of output, consumption, and investment. The model generates somewhat low volatility of hours, which is a well-known challenge for the basic neoclassical model. However, we are primarily concerned with how the information friction change the dynamics of the model and now turn to this question.

5.3.1 Exogenous Information

Before exploring the case of market-consistent information, we first examine the consequences of the information friction with an exogenous information set analogous to much of the related new-Keynesian literature (e.g. Woodford, 2002). In particular, we assume that investment choices are based on the information set

$$\hat{\Omega}^i_t = \{\hat{\theta}_{i,t-h}, \hat{s}_{i,t-h},\}_{h=0}^{\infty} \tag{57}$$

where $\hat{s}_{i,t} = \frac{1}{N}\sum \hat{\theta}_{i,t} + \nu_{i,t}$ is a signal about average productivity in the economy.[12]

Figure 2 shows impulse responses of investment and expectations to a productivity shock hitting sector *1* for the exogenous-information and full-information economy under different assumptions about the share of intermediates in the economy. Under exogenous information, other sectors learn gradually about the shock hitting sector *1*. As the right-panel shows, sectors on average have nearly completely learned the nature of the shock after five quarters. With low intermediate share, and therefore relatively weak complementarities, the dynamics of these first-order expectations essentially determine the investment response; investment adjustment is slowed only to the extent that agents gradually learn about the realization of the shock. As the intermediate share increases, however, sluggish higher-order expectations play an increasingly important role. With an intermediate share of 0.9, complementarities lead to an extremely muted and gradual response of investment to the shock.

Consistent with the impulse responses shown in Figure 2, the final line of Table 2 shows that overall volatility is much lower in the baseline model with exogenous information. In short, the model with exogenous information generates very different dynamics than the full-information model, and realistically hump-shaped responses for investment. Although we do not report them, the same patterns emerge for equilibrium output and labor in this case. These results establish that incomplete information, and higher-order expectations in particular, are at least *potentially* important for determining the paths of aggregate variables.

[12]To ensure that markets clear, we maintain the assumption that static optimality conditions continue to hold *ex post.* This is equivalent to assuming only the firm's investment decision is made with incomplete information.

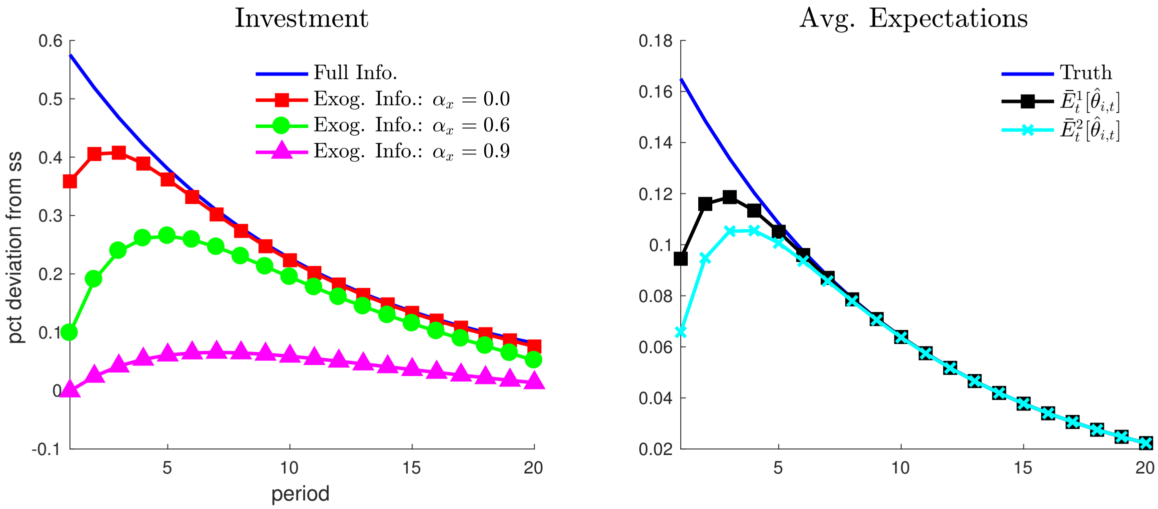


Figure 2: Investment and average expectations responses to technology shock in sector *1* under exogenous information. $\bar{E}^1_t[X_t] \equiv \frac{1}{N}\sum_{i=1}^{N} E^i_t[X_t]$ are the "first-order" average expectation of variable X_t, $\bar{E}^2_t[X_t] \equiv \bar{E}_t[\bar{E}_t[X_t]]$ are the "second-order" expectation, and so on.

5.3.2 Market-consistent Information

We now return to a version of the model in which agents observe market-based information. In particular, we consider two cases. In the first, we assume that firms observe not only their own productivity and relevant market prices, but also aggregate GDP. Consistent with our theoretical results in Proposition 2, we find that aggregate dynamics are identical to full information under the market-consistent information assumption. This result is an exact result—it is true to the numerical tolerances we set in the algorithm—and it holds regardless of the number of periods for which we assume information remains dispersed. Despite this result, sectoral dynamics are not exactly the same under the market-consistent information assumption: in the second row of table 2 sectoral investment is different at the fifth decimal place. While this difference is tiny, it highlights the fact that, theoretically, sectoral dynamics can be different under market-consistent information without any impact at all on aggregate dynamics.

What explains these results? Figure 3 shows the inference of a firm in sector *3* to the shock in sector *1*. While the firm's inference about the sectoral shocks faced by other sectors is imperfect (and indeed quite so!) it has perfectly inferred the movement in average productivity in the economy (the last figure in bottom panel.) All other firms have done the same, leaving no room for any dynamics induced by higher-order expectations (or indeed any sort of imperfect information) in the aggregate.

Next, we consider the consequence of removing GDP from the information set of firms, so that firms learn only from the relevant sectoral prices and their own productivity. The

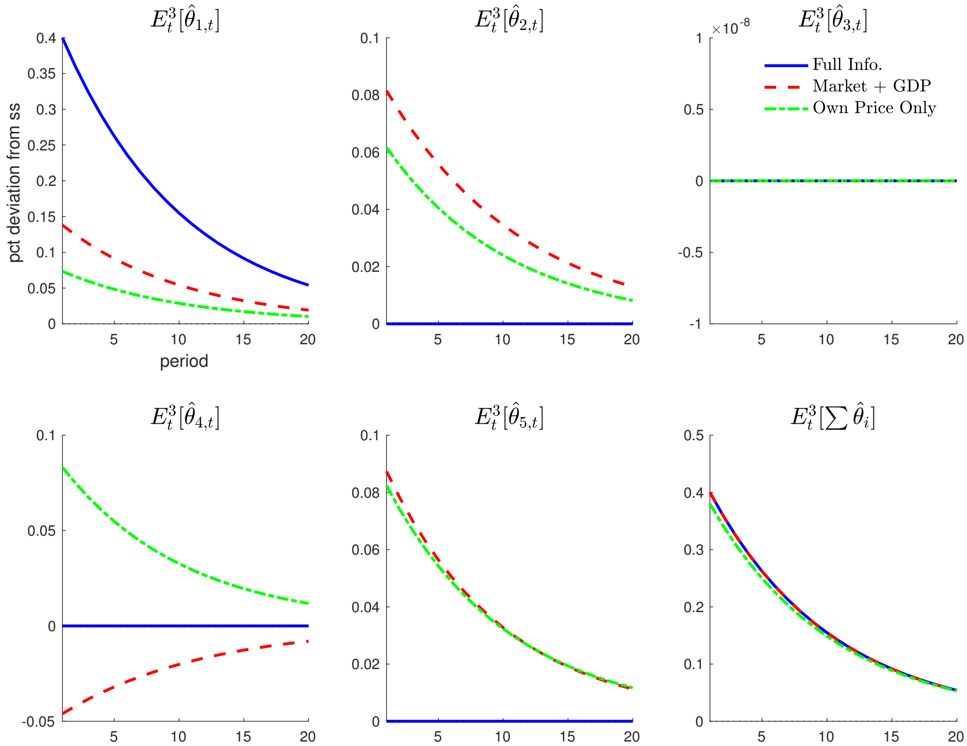


Figure 3: The inference of sector *3* in response to a sector-*1* productivity shock under alternative information assumptions.

fourth row of Table 2 shows that moments, both aggregate and sectoral, are very little changed. Despite somewhat larger sectoral mistakes, agents back out the average change in productivity so well that their inference (not reported in the figures) is visually identical to that in the full-information case. Sectoral variables do a remarkably good job of revealing the aggregate state of the economy. The nearly full revelation of aggregates, in turns, leads the aggregate consequences of the information friction to remain negligible.

To better understand how restricted the information must be to deliver substantial consequences, we consider the case in which firms observe only the price of their own good, and not that of their supplier's good. In this case, restricted information is not enough to infer the aggregate state exactly, but it still does a very good job of revealing it, as demonstrated by the green dot-dashed line in Figure 3.

Finally, to demonstrate that aggregate-informative information, even without current market information, may deliver aggregate irrelevance, we consider the case in which firms observe their own market consistent information with a one-period lag, while observing GDP contemporaneously and compare this to the case that GDP is not observed. The third row of Table 2 shows that the addition of GDP, which is a sufficient statistic for the state of aggregate productivity, once again generates aggregate moments that are identical to the full-information economy. In this case, however, sectoral quantities are dramatically

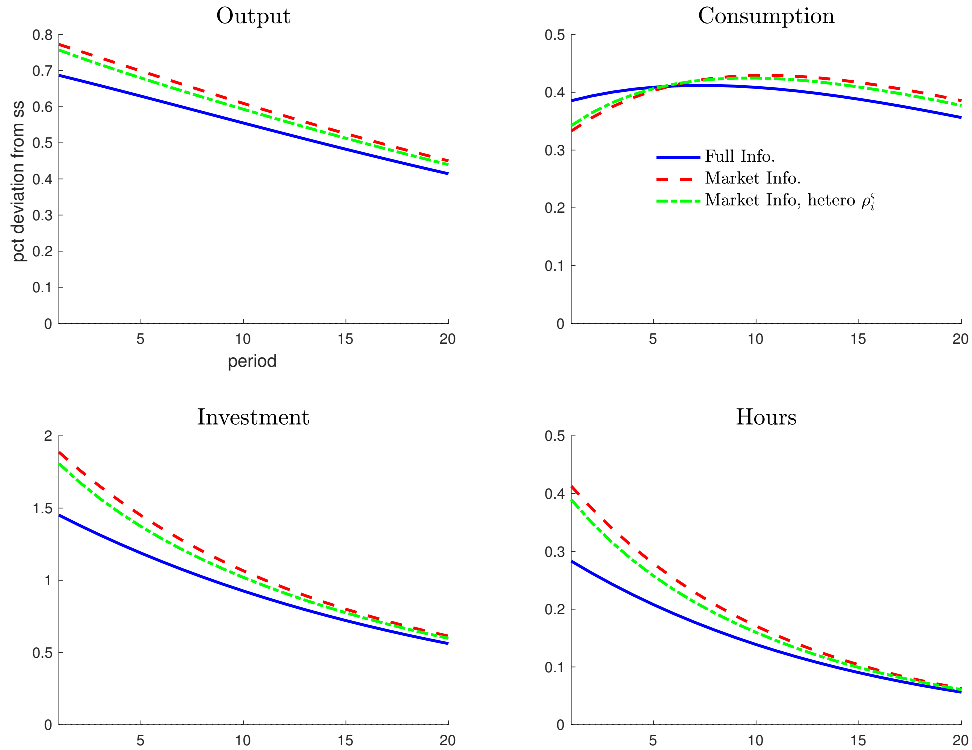


Figure 4: Aggregate impulse responses to shock to common component of technology.

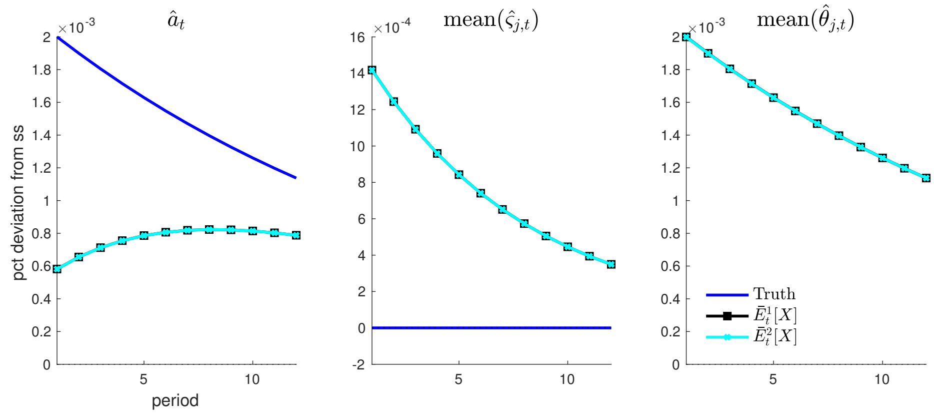
different, as demonstrated by the much greater volatility of sectoral investment in the table. This result highlights the disconnect that can occur in the economy between aggregate outcomes and the sectoral movements that generate them.

5.4 Correlated Shocks: Disentangling Aggregate and Sectoral Productivity

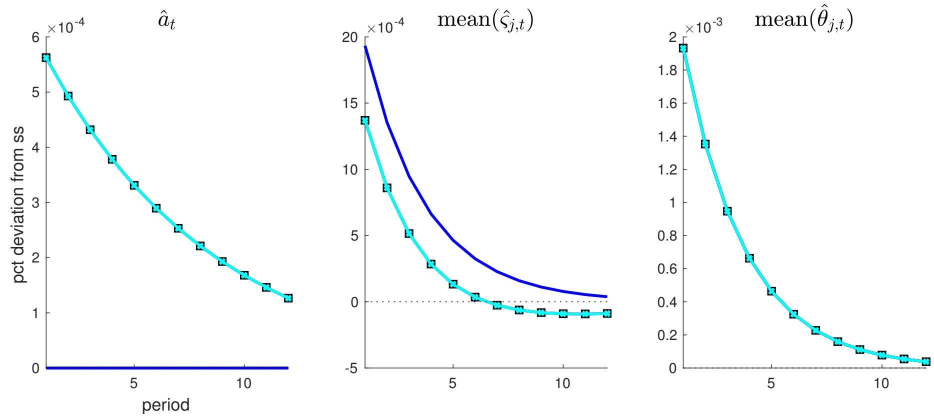
While most of the literature on sectoral linkages focuses on the effect of sectoral shocks, much of the literature on the consequences of information frictions emphasizes the difficulty agents may face in disentangling aggregate and sectoral shocks.[13] In light of Proposition 3, we calibrate the model with different persistence for the sectoral and aggregate components of productivity ($\rho_i^\varsigma = 0.70, \forall i, \rho^A = 0.95$) so that agents have a substantial incentive to distinguish between local and aggregate shocks. We then calibrate relative variances so that aggregate shocks account for around 50% of aggregate fluctuations under full information, in line with Foerster et al. (2011).

Figure 4 report impulse response to an aggregate technology shock under alternative information assumptions. The figure shows that, relative to full information, the restriction to market-based information has a modest effect on responses to the aggregate shock.

[13]For some examples, see Lorenzoni (2009), Graham and Wright (2010), and Acharya (2013).



(a) Aggregate shock



(b) Sectoral shock

Figure 5: Expectations responses to aggregate and sectoral productivity shocks.

Moreover, this effect is precisely the opposite of the slow adjustment that arises in the case exogenously dispersed information.

The overreaction of investment arises in this case from a common misperception about future productivity. Market information continues to reveal aggregate productivity, but firms are uncertain about whether that average productivity is driven by a coincidence of (more temporary) sectoral shocks or by a (more permanent) aggregate shock. Since the optimal response to less persistent shocks in our economy is to raise saving and investment, firms' inference that at least part of the aggregate shock is due to sectoral disturbances leads them all to raise investment above the full information level. Though we do not report it, the effect of incomplete information on *unconditional* second moments turns out to be tiny, since the "over reaction" in response to aggregate shocks is offset by an "under reaction" to sectoral shocks.

These findings confirm Proposition 3 regarding the disappearance of information dispersion. Panel (a) of Figure 5 shows that in response to the aggregate shock, first-order and higher-order expectations of the shock are perfectly aligned, i.e., there is no disagreement about the aggregate in the economy. Panel (b) of Figure 5 shows that, in response to a sector-specific shock, agents also share identical beliefs about which shocks are driving the economy. Without disagreement, however, dispersed information cannot play a role in driving aggregate dynamics.

Finally, to ascertain the importance of the assumption of a common persistence of sectoral shocks, we consider a version of the model in which the autocorrelation of sectoral shocks is the same *on average*, but exhibits heterogeneity across sectors. Specifically, we assume that the values of ϕ_i^ς are given by $\{0.95, 0.85, 0.75, 0.65, 0.55, 0.45\}$, so that average sectoral persistence is 0.7, as before. In this case, agents generally disagree about the aggregate and sectoral components of the shocks hitting the economy, but again, these disagreements tend to cancel out on average, leaving aggregate responses (shown in Figure 4) to be very close to those in the economy with a common sectoral persistence parameter.

5.5 Additional Robustness

We performed several additional robustness exercises, all of which confirm the basic results in this section. These included specifying the economy with several alternative types of preferences, incorporating sector-level demand rather than productivity shocks, including either capital or investment adjustment costs, considering versions of the economy with a real risk-free bond that trades across sectors, and a version of the economy populated by a representative household with full information, while maintaining diverse information at the firm level. In all of these cases, we recover numerically exact aggregate irrelevance when the main conditions of Proposition 2—circulant IO structure and aggregate informative information—are met. Even in cases in which no irrelevance result applies, we always find qualitatively tiny differences between the full- and incomplete-information economies.

6 Information Transmission in Model Calibrated to US Data

In this section, we calibrate the model to match US data on the sectoral input-output structure and the empirical measures of sectoral total-factor productivity. In doing so, we simultaneously relax all of the assumptions underlying Propositions 1 through 3 regarding production shares, the input-output matrix, and the shock processes. With the assumptions of our theorems strongly violated, information frictions have the potential to play a substantially larger role in explaining aggregate dynamics. Our results, however, show that aggregate dynamics in the calibrated diverse-information model remains remarkably close

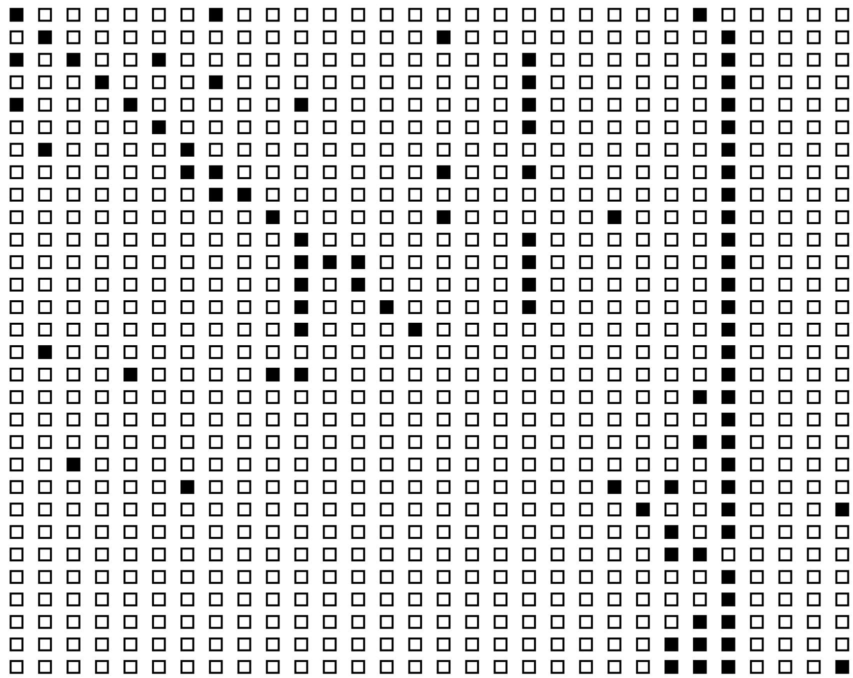


Figure 6: Sparsity of the US input-output table in the 30 Jorgenson et al. (2013) sectors.

to those under complete information.

6.1 Empirical Input-Output Linkages

We start by calibrating the intermediate shares of each sector in the economy to match the empirical input-output tables for the US economy. The raw data for these tables come from the detailed benchmark table for the year 2002 available from the Bureau of Economic Analysis, at http://www.bea.gov/industry/iedguide.htm#io. At this fine level of disaggregation, in which the US economy is divided into roughly 450 different sectors, the input-output table is quite sparse, with less than 2% of entries being non-zero. Ideally, we would proceed with this completely disaggregated input-output structure. This is not possible, however, both because numerical limitations prevent us from solving the model at such a disaggregated level and because no analysis of sectoral productivity exists at such a refined level.

In order to proceed, we aggregate the input-output tables to correspond with the 30 Jorgenson et al. (2013) industries, according to correspondences provided by those authors.[14] Very few entries in the resulting partially aggregated input-output matrix are strictly zero; however many entries remain relatively very small. Thus, in our calibration, we treat as zero any input that accounts for less than 4% of gross output in a particular industry, reallocating

[14] Jorgenson et al. (2013) describe 32 sectors. However two of those sectors, that of home production and non-comparable imports, do not map well into the model. For these reasons, we exclude them from our calibration.

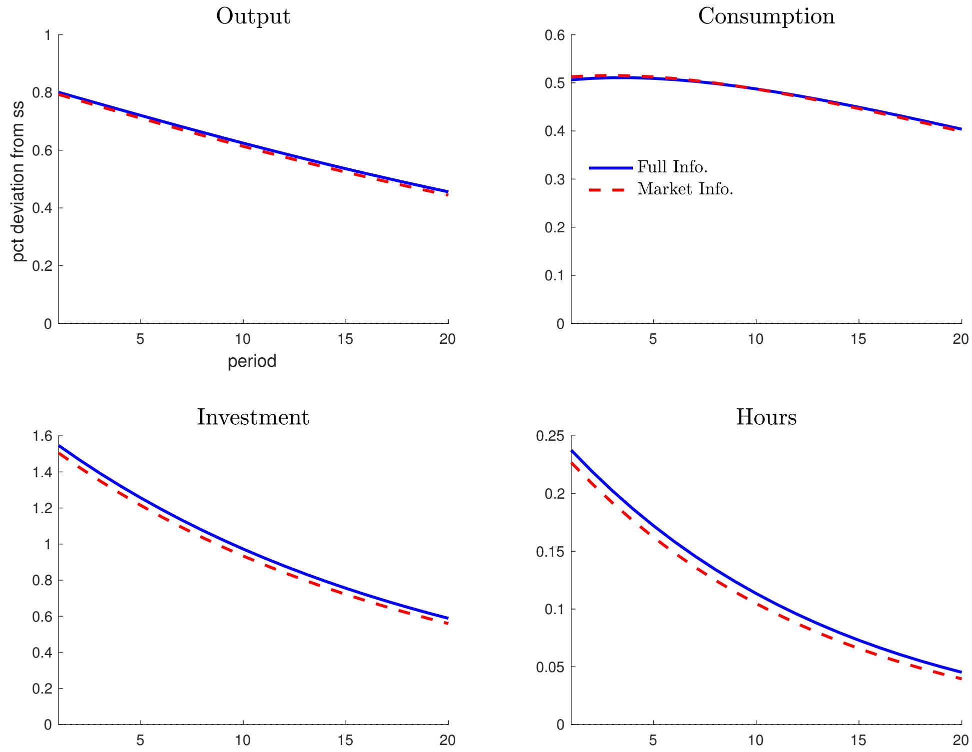


Figure 7: Impulse responses to an aggregate technology shock for the full- and market-consistent information models calibrated to US data.

that share proportionally to inputs with larger initial shares to keep the total intermediate share constant. Notice that if we had chosen a lower cutoff then firms in our economy would observe yet more prices, further limiting the potential for incomplete information to matter.

Figure 6 visually represents the structure of the resulting-input output matrix, with dark squares representing non-zero entries in the input-output table. Roughly 10% of all entries are non-zero, and the matrix is highly diagonal; off-diagonal sparsity is substantially higher. The matrix is also highly asymmetric, with the sector "renting of machine and equipment, and other business services" constituting a non trivial input in nearly every other industry. In short, the input-output matrix is very different from the stylized symmetric formulation used in our earlier examples.

6.2 Sectoral Productivity

To calibrate the process for the aggregate and idiosyncratic TFP shocks, we proceed by estimating the unrestricted factor model for sectoral productivity found in equations (15) - (17). To do this, we treat equation (15) as a measurement equation, with $\hat{\varsigma}_{i,t}$ and $\hat{a}_t$ as unobserved components, and estimate the parameters $\{\rho^A, \rho_i^{\varsigma}, \mu_i, \sigma_i\}$ using the sectoral TFP measurements of Jorgenson et al. (2013) and Bayesian methods with flat priors. Table 3 reports the estimated autocorrelation coefficients, showing that indeed there is substantial

Table 3: Estimated parameters for sectoral TFP factor model.

	ρ_i	σ_i	μ_i
aggregate tfp	0.95	0.01	
sectoral mean	0.95	0.03	1.27
agriculture, hunting, forestry and fishing	0.87	0.05	0.63
mining and quarrying	0.98	0.04	2.32
food, beverages and tobacco	0.96	0.04	1.14
textiles, leather and footwear	0.92	0.03	-0.18
wood and products of wood and cork	0.97	0.03	-0.66
pulp, paper, printing and publishing	0.98	0.02	1.67
chemical, rubber, plastics and fuel	0.85	0.02	5.92
coke, refined petroleum and nuclear fuel	0.96	0.15	13.85
chemicals and chemical products	0.98	0.03	4.31
rubber and plastics	0.90	0.03	1.97
other non-metallic mineral	0.86	0.03	1.60
basic metals and fabricated metal	0.95	0.02	1.09
machinery, not elsewhere classified	0.98	0.04	0.80
electrical and optical equipment	1.00	0.04	-0.41
transport equipment	0.92	0.04	1.84
manufacturing not elsewhere classified; recycling	0.96	0.03	1.07
post and telecommunications	0.97	0.02	-0.46
construction	1.00	0.02	0.58
sale, maintenance, and repair of vehicles; sale of fuel	0.92	0.04	1.60
wholesale trade and commission trade, excl. vehicles	0.94	0.03	1.16
retail trade, excl. vehicles; repair of household goods	0.93	0.03	1.40
hotels and restaurants	0.99	0.02	0.27
transport and storage	0.94	0.02	0.51
post and telecommunications	0.97	0.02	-0.44
financial intermediation	0.98	0.03	-0.45
real estate, renting and business activities	0.98	0.01	-0.13
real estate activities	0.97	0.02	-0.30
renting of manu. & equip. and other business activities	0.94	0.02	0.08
public admin and defense; social security	0.97	0.02	-0.23
education	0.98	0.02	0.16
health and social work	0.97	0.02	-0.20
other community, social and personal services	0.98	0.01	0.11

Note: Table provides posterior median estimates for each parameter. Estimation uses flat priors for all parameters. AR parameters constrained to be strictly less than one. Aggregate refers to the parameters of the aggregate TFP process. Sectoral mean provides the mean over median posterior values of all sectors. Standard errors and modal values are available in the appendix. Sector names modified slightly from Jorgenson at al. (2013) data file for clarity.

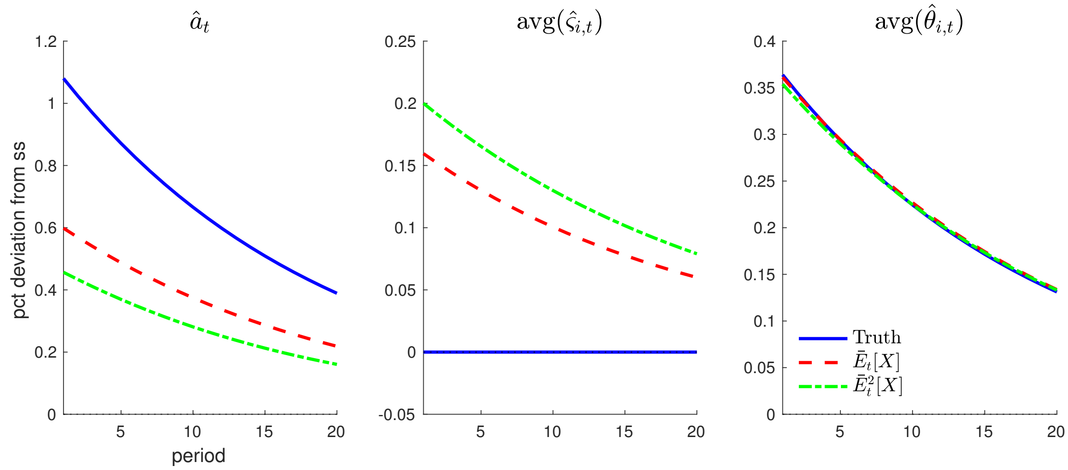


Figure 8: Average expectations responses to aggregate productivity shock in the model calibrated to US sectoral data. The aggregate variables $avg(\hat{\varsigma}_{i,t})$ and $avg(\hat{\theta}_{i,t})$ are weighted according to each sector's contribution gross output.

sectoral heterogeneity in the persistence of shocks. Nevertheless, the average estimate of sectoral persistence is quite close (identical to two decimals) to the estimated persistence of the aggregate component, suggesting that firms have relatively little incentive to disentangle aggregate and idiosyncratic shocks. For completeness, the remaining columns show estimated sectoral variances and the corresponding weights on the aggregate component. These estimated values also show substantial heterogeneity across sectors. We set all parameters not related to the input-output structure and sectoral productivity processes at their baseline values in Table 1.

6.3 Effects of Incomplete Information

Figure 7 plots the impulse responses of the model under full and market-based information. The impulse responses of the realistically calibrated economy are almost entirely unaffected by the presence of incomplete information. Indeed, the similarity here is even stronger than that in our stylized economy with an aggregate/sectoral inference problem, depicted in Figure 4.

To better understand this striking result, Figure 8 plots expectations responses for the exogenous processes driving the economy. The first two panels show that, in this asymmetric environment, endogenous information does a rather poor job of revealing the arrival of the common shock to firms in the economy. Even 12 quarters after the shock, firms mistakenly attribute, on average, more than half of the shock to idiosyncratic shocks rather than the common component (dashed red line versus solid blue.) Moreover, there is substantial *dispersion* of information about the contribution of these two sources, which can be seen by

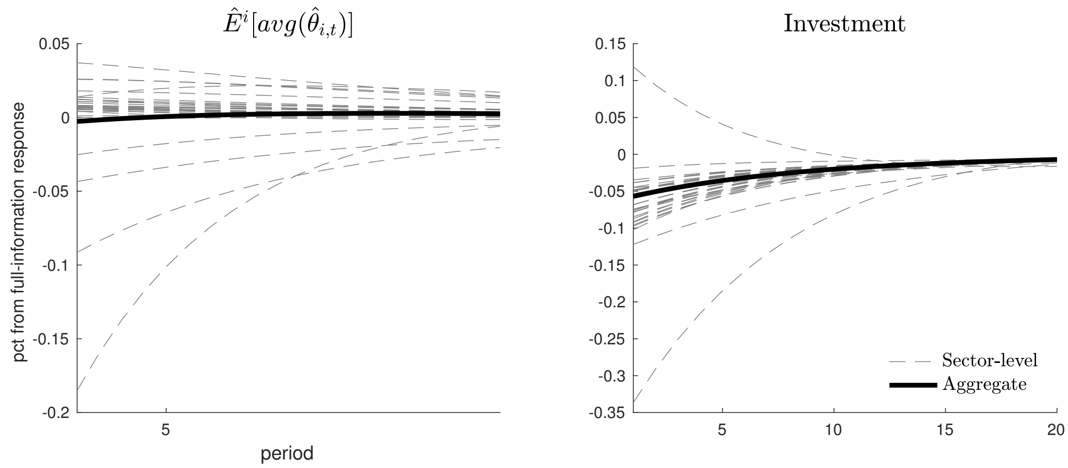


Figure 9: Sector-level expectations and investment responses, relative to full information, after aggregate productivity shock in the model calibrated to US sectoral data.

noticing the relatively large gap between second-order average expectations (dot-dash green line) and first-order average expectations (dashed red line.) Nevertheless, firms' beliefs about aggregate productivity, which combines both common and sectoral shocks, are remarkably close to the truth on average. While firms cannot distinguish the source of the shock, market-based observations lead average expectations to track average productivity almost perfectly.

Figure 9 provides additional information on the dispersion within average beliefs. The first panel plots sector-level mistakes regarding the state of aggregate productivity. The figure shows that even regarding current aggregate productivity, there now exists a small but non trivial degree of dispersion in beliefs. Nevertheless, sector-level mistakes are closely centered at zero, precluding the pattern of increasingly sluggish higher-order beliefs that slows aggregate responses in the exogenous informational economy. In short, expectational mistakes regarding the average are small, consistent with Proposition 1, and negatively correlated, consistent with Proposition 2.

The second panel of Figure 9 shows investment responses at the sector level, relative to their full-information counterparts. Like beliefs about productivity, sectoral investment exhibits modest but non trivial dispersion. Compared to the case for productivity expectations, however, investment exhibits a small bias relative to the full-information economy. The small size of the bias here reflects our estimation result that average persistence of sectoral productivity is very close to that of the common component of productivity. As suggested by Proposition 3, the behavior of the economy closely approximates the behavior of a common-knowledge, representative-agent economy in which the two components of aggregate productivity have very similar persistence. While agents disagree over long periods about the cause of the price changes they see in their own markets, these disagreements

Table 4: Explanatory power of market information for private and aggregate conditions in the full information economy.

	Sectoral Inv.	**GDP**
Avg R^2	0.991	0.979

have small effects on actions because agents in each sector expect those changes to last the same amount of time *regardless of their source.*

Table 4 provides a final perspective on the practical relevance of Proposition 1, which showed in a special case that market consistent information was perfectly informative about both optimal sectoral responses and the aggregate state of full information of the economy. We revisit this result quantitatively by projecting the full information optimal sectoral investment choice and aggregate output on each sector's market-based information. The table reports the (size-weighted) average share of the variance of these optimal choices that can be explained by market based information and the result is striking: market-based information explains 99% of the optimal investment response and 98% of aggregate output. In practice, market consistent information is extremely close to being *both* action and aggregate informative.

7 Conclusions

Here we have explored an environment of strategic interactions among firms in which exogenously dispersed information leads to large consequences for aggregate dynamics, but learning through market prices virtually eliminates the effect of incomplete information. This is true even though sectoral dynamics can change, sometimes substantially, and no law of large numbers is available. In one respect, this paper makes the cautionary point that informational asymmetries and strategic interdependence, the two key ingredients in much of the related literature, do not guarantee an important role for information. We believe that the key assumption driving this difference—that firms condition their investment choices on their market-based information—is realistic. More generally, we have argued that general equilibrium places important restrictions on expectations conditioned on endogenous information, many of which are independent of the precise details of the agents' information set. Our analytical results offer some avenues for breaking these results, and thereby generate an important role for information frictions. However, our quantitative results suggest that even when exact irrelevance fails to hold, the plausible quantitative implications remain small.

8 Acknowledgements

The research leading to these results has received financial support from the European Research Council under the European Community's Seventh Framework Program FP7/2007-2013 grant agreement No.263790. Thank you to Ethan Struby for excellent research assistance. The authors thank the editor, Vincenzo Quadrini, two anonymous referees, along with Philippe Andrade, George-Marios Angeletos, Susanto Basu, Sanjay Chugh, Patrick Fève, Gaetano Gaballo, Christian Hellwig, Jean-Paul L'Huillier, Martí Mestieri, Jianjun Miao, Kristoffer Nimark, Franck Portier, Ricardo Reis, Robert Ulbricht, Michael Woodford, and innumerable seminar/conference participants for helpful comments and suggestions.

References

Acemoglu, D., V. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The Network Origins of Aggregate Fluctuations. *Econometrica 80*(5), 1977–2016. 2

Acharya, S. (2013). Dispersed Beliefs and Aggregate Demand Management. Working paper, University of Maryland. 13

Basu, S. (1995). Intermediate Goods and Business Cycles: Implications For Productivity and Welfare. *The American Economic Review 85*(3), pp. 512–531. 11

Benhabib, J., P. Wang, and Y. Wen (2015). Sentiments and aggregate demand fluctuations. *Econometrica 83*(2), 549–585. 3

Carvalho, C. and J. W. Lee (2011). Sectoral Price Facts in a Sticky-Price Model. *FRB of New York Staff Report* (495). 11

Chahrour, R. and G. Gaballo (2018). Learning from Prices: Amplification and Sentiments. Boston College Working Paper 873. 3

Dupor, B. (1999). Aggregation and Irrelevance in Multi-sector Models. *Journal of Monetary Economics 43*(2), 391–409. 2, 1, 4.1, 4.1

Foerster, A. T., P.-D. G. Sarte, and M. W. Watson (2011). Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy 119*(1), 1–38. 5.4

Graham, L. and S. Wright (2010). Information, Heterogeneity and Market Incompleteness. *Journal of Monetary Economics 57*(2), 164 – 174. 1, 3.4, 9, 13

Hellwig, C. and V. Venkateswaran (2009). Setting the Right Prices For the Wrong Reasons. *Journal of Monetary Economics 56*(Supplement 1), S57 – S77. 1

Hellwig, C. and V. Venkateswaran (2014). Dispersed Information, Sticky Prices and Monetary Business Cycles: A Hayekian Perspective. Working Paper. 1, 4.3.1

Horvath, M. (1998). Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks. *Review of Economic Dynamics 1*(4), 781 – 808. 2

Horvath, M. (2000). Sectoral Shocks and Aggregate Fluctuations. *Journal of Monetary Economics 45*(1), 69 – 106. 2, 5.1

Jorgenson, D. W., M. S. Ho, and J. D. Samuels (2013). A Prototype Industry-Level Production Account For the United States, 1947-2010. Working Paper. 1, 6, 6.1, 14, 6.2

Long, J. B. and C. I. Plosser (1983). Real Business Cycles. *Journal of Political Economy 91*(1), pp. 39–69. 2, 3

Lorenzoni, G. (2009, December). A Theory of Demand Shocks. *American Economic Review 99*(5). 5, 13

Lucas, R. E. (1972). Expectations and the Neutrality of Money. *Journal of Economic Theory 4*(2), 103 – 124. 3

Melosi, L. (2014). Estimating Models With Dispersed Information. *American Economic Journal: Macroeconomics 6*(1), 1–31. 5

Moro, A. (2012, 4). Biased Technical Change, Intermediate Goods, and Total Factor Productivity. *Macroeconomic Dynamics 16*, 184–203. 5.1

Nakamura, E. and J. Steinsson (2010). Monetary Non-Neutrality in a Multisector Menu Cost Model. *Quarterly Journal of Economics 125*(3), 961 – 1013. 11

Nimark, K. (2008). Monetary policy with signal extraction from the bond market. *Journal of Monetary Economics 55*(8), 1389–1400. 3.4

Rondina, G. and T. B. Walker (2017). Information Equilibria in Dynamic Economies With Dispersed Information. Working paper, Indiana University. 9

Sims, C. A. (2003). Implications of Rational Inattention. *Journal of Monetary Economics 50*(3), 665 – 690. 3.4

Townsend, R. M. (1983). Forecasting the Forecasts of Others. *The Journal of Political Economy 91*(4), 546–588. 5.1

Vives, X. (2017). Endogenous Public Information and Welfare in Market Games. *The Review of Economic Studies 84*, 935–963. 3

Woodford, M. (2002). *Knowledge, Information and Expectations in Modern Macroeconomics*, Chapter Imperfect Common Knowledge and the Effects of Monetary Policy. Princeton, NJ: Princeton University Press. 5, 5.3.1

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press. 5.1, 5.2